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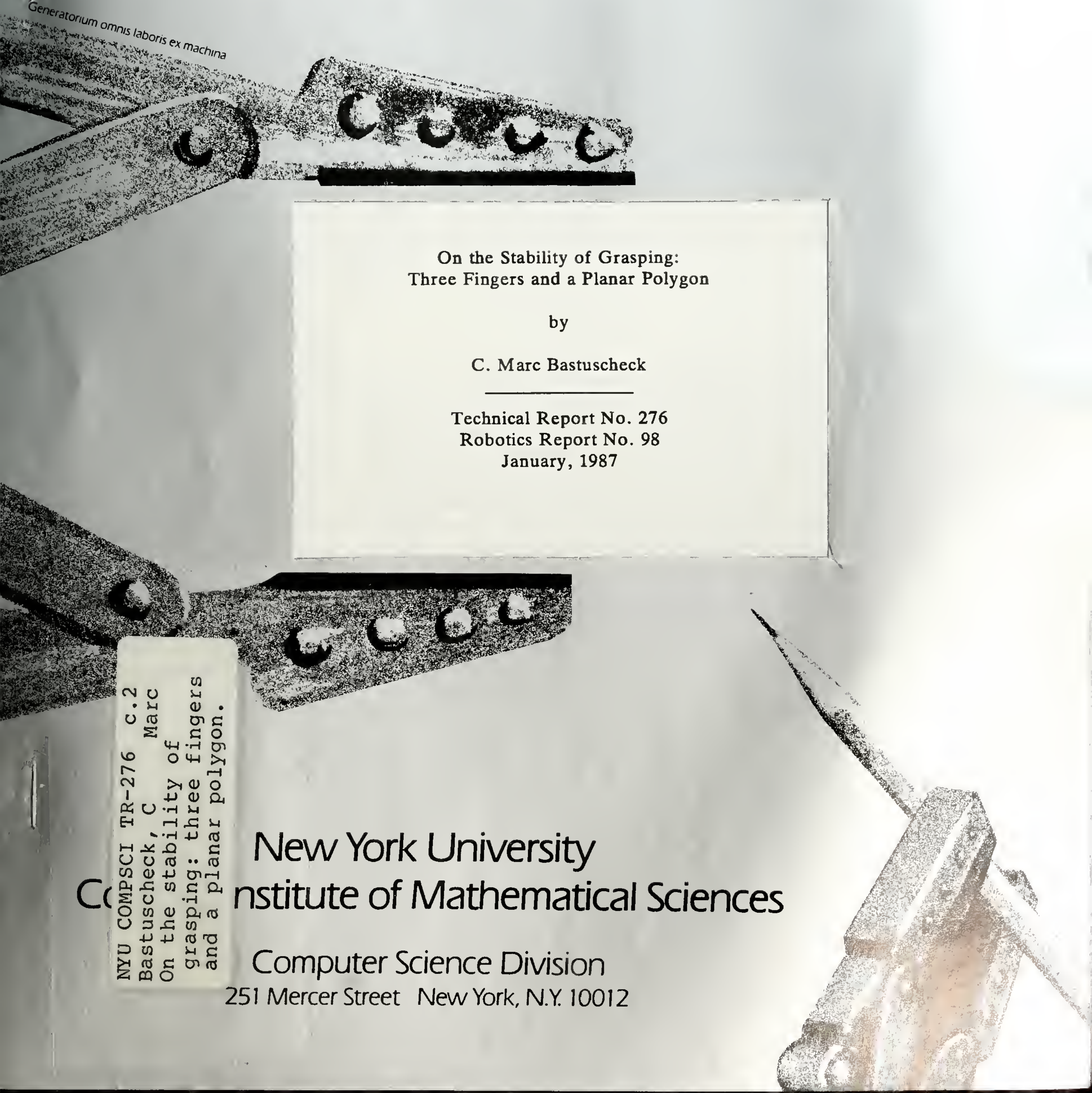

On the Stability of Grasping: Three Fingers and a Planar Polygon

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C. Marc Bastuscheck

Technical Report No. 276
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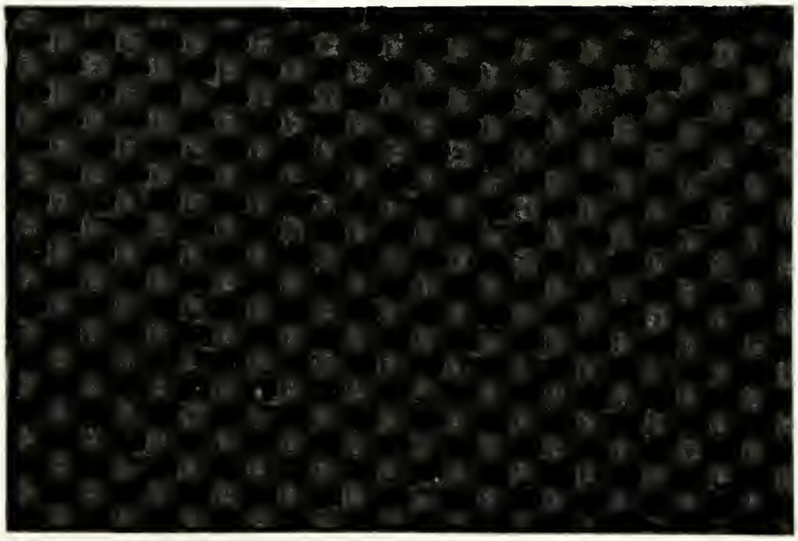

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On the Stability of Grasping: Three Fingers and a Planar Polygon[1]

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ABSTRACT

Grasping an object can be accomplished with very little information about the shape of the object. The approach to a stable grasp is examined carefully for the 2D case of a triangle being grasped by 3 fingers with no effects of friction or mass. The motion of the triangle in contact with two fixed fingers and one moving finger is determined, and a large class of motions is found to result in an equilibrium grasp. The results easily generalize to planar polygons, are expected to extend to real fingers grasping plate-like polygonal objects in the presence of mass and friction. Optimum grasping points for poorly known objects are given as those with the best toleration of error.

1. Introduction

The problem of grasp planning is often approached with tacit assumptions that the object to be grasped is perfectly known, and that the end-effector or hand can be positioned with perfect control of force and location of every element in contact with the object. In many robotic applications, however, only a rough idea of object shape and location may be available, e.g. provided by a vision system. It is shown in this paper that uncertain information can be sufficient to ensure successful grasping because an object tends to move towards a more stable configuration under certain grasping conditions. This behavior can be exploited in procedures for initial object acquisition or for pick-and-place operations, since the identity and/or orientation of the grasped object can be sensed after acquisition and subsequent robot motion appropriately adjusted.

In this paper issues involved in the stable approach to a grasp are investigated theoretically. The problem is solved entirely for a triangle constrained to move in a plane in contact with two fixed point fingers when the effects of mass and friction are ignored. The motion of the triangle is given as well as a description of paths which the moving finger may follow which result in an equilibrium grip. The grasping process is found to be quite stable, and remains stable more generally: cases are considered in which fingers are not points, mass and friction are present, and more general objects are grasped.

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[1] Work on this paper has been supported by Office of Naval Research Grant N00014-82-K-0381, National Science Foundation CER Grant DCR-83-20085, and by grants from the Digital Equipment Corporation and the IBM Corporation.

The theory developed here pays little attention to actual mechanical implementations. It is assumed that fingers can be controlled independently, that a finger behaves as a spring acting on a point, and that the compression of the spring lies in the same line as its force. This corresponds quite well to the mechanism of the (planar) Four Finger Manipulator at New York University, and the procedures suggested here can be implemented easily on this machine. Implementation on three dimensional hands may be more difficult because of the required changes of direction of applied forces during the grasping process.

Many aspects of the problem of grasping objects using computer controlled machines have been explored, including theoretical stability of grasps, use of friction, applications with specific hands of 2, 3, or more fingers, and computational complexity of grasp planners. The following recent papers have been helpful in shaping the presentation of the ideas which follow, and may be consulted for additional references to the literature. Salisbury and Craig [SC82] consider generally issues of kinematics and force in the design of articulated hands. Hanafusa and Asada [HA77] consider grasping with a three-fingered hand having elastic fingers fixed at 120 degree intervals. Baker et al. [BFG85] examine the stability of three-finger planar grips in detail and discover that some grips which intuitively would seem to be stable can in fact be unstable, giving as examples spring-like fingers gripping a circle, and the hand of Hanafusa and Asada gripping some polygons. Holzmann and McCarthy [HM85] have investigated frictional forces involved in gripping 3D objects with 3 fingers, and (with Abel) planar objects with 2 fingers [AHM85]. Nguyen has explored procedures for generating stable force closure grips having controllable compliance both with [N86a] and without [N86b] friction.

2. Frictionless Equilibrium Grasp

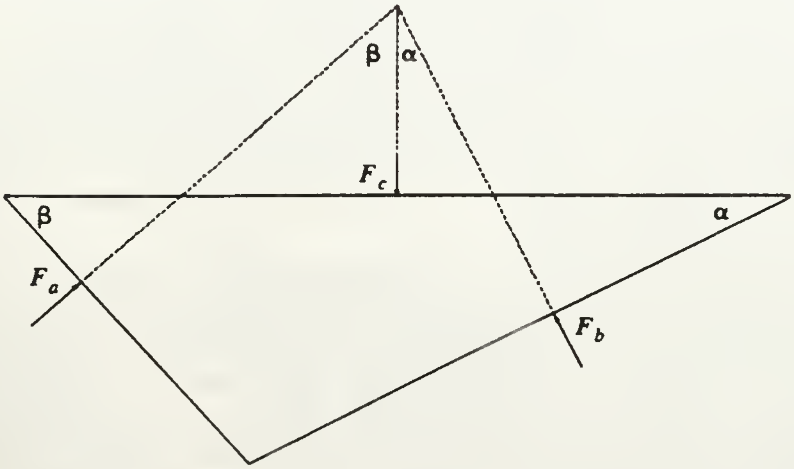


Figure 1. Fingers at the arrows establish an equilibrium grasp.

An equilibrium grasp of an object is one in which no net force or torque is exerted on the object. In the case of three fingers in two dimensions this means that the lines of force of the fingers must meet in a point, as shown in Fig. 1. The forces applied by the fingers are perpendicular to the sides of the triangle, and have ratios

$$F_a = \frac{\sin\alpha}{\sin(\alpha+\beta)} F_c \qquad F_b = \frac{\sin\beta}{\sin(\alpha+\beta)} F_c. \tag{1}$$

where the forces F and angles are as shown in Fig. 1. Thus from the directions or ratios of the observed forces the shape of a grasped triangle can be determined.

An equilibrium grasp is stable if, when the base of spring-like fingers is stationary and the object is slightly perturbed, the object returns to its equilibrium position. In the case of a triangle being held by three point, spring-like fingers with no friction it is easily shown that an equilibrium grip is stable against infinitesimal perturbations ([BFG85],[N86b]). However, this stability of a grasp is quite different from the stable *approach* to a grasp discussed below. In this paper the object may make large motions during the grasping operation; the final grasp and the limits of the allowed motions will be determined.

3. Grasping a Triangle

In this section it is assumed that fingers act as points without friction, the triangle moves on a plane without friction, effects of mass are negligible, and that two fingers are fixed and in contact with two sides of the triangle while a third finger moves in contact with the third side of the triangle. It will be shown that for many paths of the moving finger the triangle will move in a completely determined manner until an equilibrium grip is established.

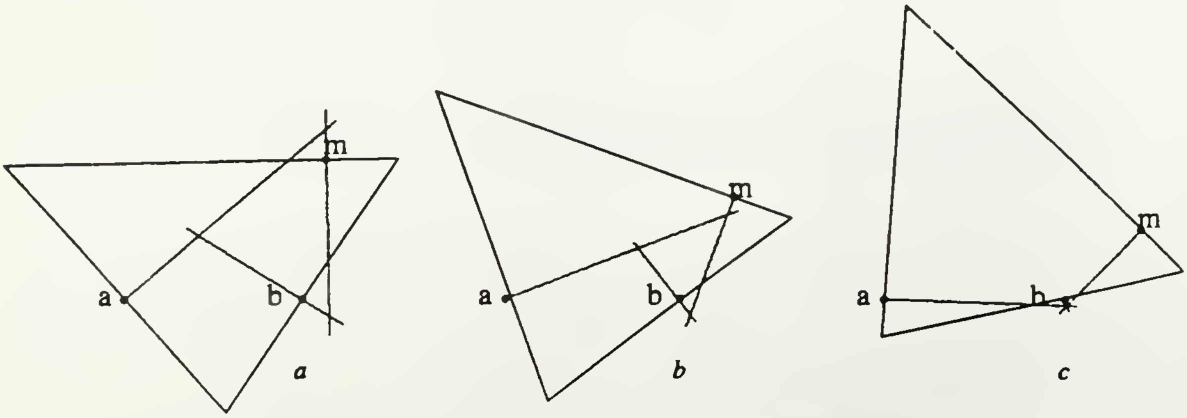


Figure 2. Triangle rotates in contact with fixed fingers a and b to a stable grip.

An intuitive understanding of the process follows from examination of Fig. 2. Here fingers a and b are fixed against two sides of a triangle and finger m moves against the third side. The dotted lines are normals to the sides of the triangle at the finger contact points; if the triangle had mass or friction against the plane these would be lines of force exerted by the fingers. In representations 2a and 2b these lines do not meet at a point, and thus do not represent an equilibrium grip. As finger m moves against its side the triangle moves, and the effect of the motion of the triangle is generally to decrease the area of the triangle formed by the normals. The motion of the triangle at any instant is a rotation about the intersection of the normals at a and b since motion of a side into a fixed finger is impossible. The sides of the triangle slip along the fingers. As finger m continues to move the representation of Fig. 2c is reached: the normals meet in a point, and further motion stops. Forces appear, and distribute themselves according to eqn. 1. In the following paragraphs the motion of the triangle, the final location of the intersection of the normals and the final position of the moving finger *m* will be determined quantitatively.

An equilibrium grip fails to develop only when any one of the fingers loses contact with its side. This is equivalent to the observation that the intersection of the normals in an equilibrium grip must lie in a region *P* which is bounded by perpendiculars from the ends of all edges of the triangle. This region is shown for one triangle in Fig. 3. Note that region *P* exists in the frame of reference of the triangle and rotates with it. Region *P* always includes part of the triangle and always extends beyond the triangle. When the triangle is obtuse (as in Fig. 3) it cannot include all of the triangle and has only four sides; with acute triangles it is bounded by all six perpendiculars, as in Fig. 4.

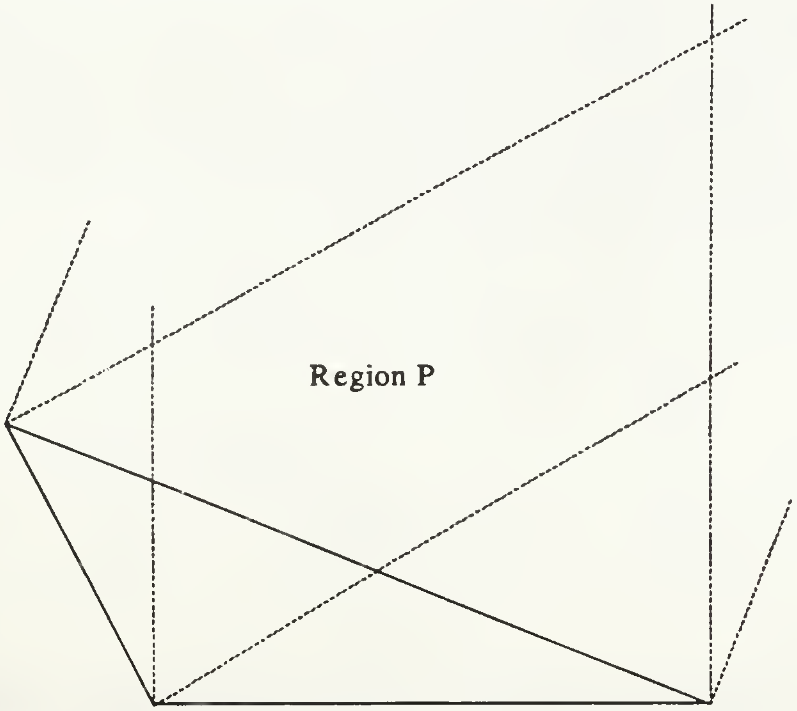


Figure 3. Region P is bounded by normals to the ends of the triangle edges.

The motion of the triangle and limits to the motion of the moving finger can be quantified if the triangle is known. In the following the main results will be simply stated with discussion to clarify their effects. Proofs of all results are given in the appendix; for the convenience of the reader each result is given as a numbered theorem so the corresponding proof can be found easily. Emphasis here is on application to grasping, since the theorems are trivial, presumed to have been published long ago (somewhere), or both.

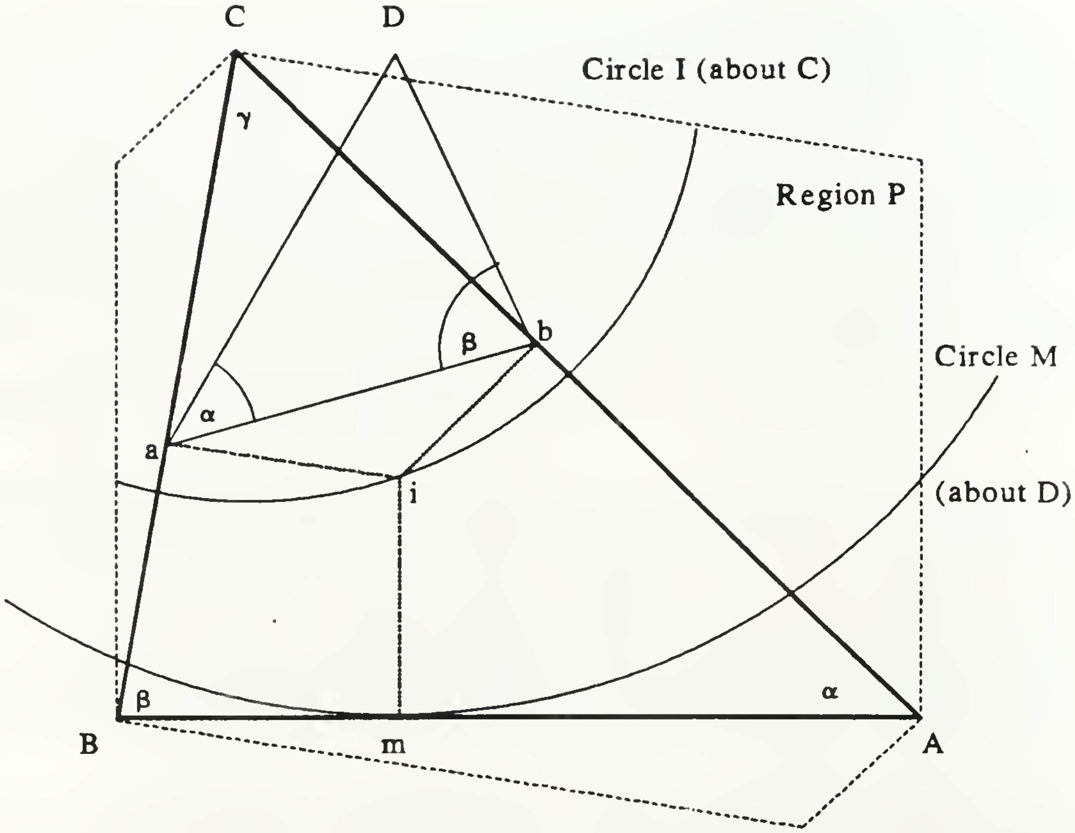


Figure 4. Construction showing the primary results. Moving finger m, fixed fingers a and b; normals are dotted, boundary of region P is dashed.

The letters in the following theorems refer to Fig. 4, which shows a triangle with vertices A, B, and C having angles α, β, and γ respectively. The fixed fingers are modeled using points a and b which lie in sides BC and AC respectively, holding vertex C between them. Note that some features in Fig. 4 are in the frame of reference of the triangle and some are in the frame of reference of the fixed fingers.

Theorem 1: The vertex C of the triangle is constrained to move on a circle (see Fig. 6) which includes the current position of the vertex and the fixed fingers a and b. This circle is useful for determining the motion of the triangle, and also for proof of the following theorem. It is easily shown that motion of C about the center of this circle by θ corresponds to rotation of the triangle by $\theta/2$.

Theorem 2: The locus of points m which are the location of the moving finger when it reaches an equilibrium grip of the triangle ABC held against two fixed fingers a and b is a circular arc M about a point D in the frame of the fixed fingers; D lies at the intersection of the line making angle β with line ab at b and the line making angle α with line ab at a and has radius $AC \sin\alpha$ or (equivalently) $BC \sin\beta$. This means that if the finger m moves from any starting location toward this arc through side AB an equilibrium grip is assured. The circle exists in the frame of reference of the fixed fingers. However, there are ends to the useful portion of this arc which correspond to fingers falling off sides, and these are not immediately known in this frame of reference. (The ends are drawn correctly in Fig. 4.)

Theorem 3: The locus of points i which are the intersection of the normals to the sides of the triangle at a and b is a circle I about the vertex C; the radius of this circle is $L/\sin\gamma$, where L is the distance between the fixed fingers. The circle exists in the frame of reference of the triangle. Since the intersections of the normals in equilibrium grips must lie on it and also within the region P given above (which also exists in the frame of reference of the triangle), there is a clearly defined arc (under some conditions with a gap) where the normals may intersect in an equilibrium grip.

Theorem 4: The motion of the point i (intersection of normals at fixed fingers) along circle I about C corresponds to the measure of the rotation of the triangle with respect to the fixed fingers.

Theorem 5: The motion of the point m along the circle M about D corresponds to the measure of the rotation of the triangle with respect to the fixed fingers.

Theorems 3, 4, and 5 together determine those portion(s) of M at which the moving finger may establish an equilibrium grip, since points on I map to points on M. The limit points on M can be found as follows: in the starting configuration determine points i and m, the circles I and M and the region P. Determine the endpoints of those portions of I which lie inside region P, and the arcs about C from i to these endpoints. Mark corresponding arcs about D on M from m. Any motion of the moving finger through the third side of the triangle to an allowed region of M will result in a stable grip. As suggested in Fig. 4, the allowed arc is usually large, and equilibrium grips are easily obtained.

This, then, is the **main theoretical result**: that for reasons which are simply geometric, motion of a finger against the third side of a triangle which is in contact with two fixed fingers on two sides will result in a stable grasp as long as the finger moves toward an easily determined circular arc. Only when a finger loses contact with its side does a grasp fail to result, and the conditions when this happens are known. The remainder of this paper considers realistic applications of this result.

4. Grasping imperfectly known triangles

The above results were derived assuming that the triangle and finger spacing and locations were perfectly known. Grasping an imperfectly known triangle could be approached by estimating worst case errors and determining allowed arcs M for each, then driving the moving finger so as to intersect all of them. However, in view of the large region of stability observed, it appears to be sufficient to move the third finger through the third side toward the center of the allowed arc calculated for an observed triangle and finger spacing, and let the triangle slip to adjust for small errors. After the grip is established the shape of the triangle can be determined from the ratios of the (vector) forces on the fingers (see eqns. 1).

Grasps for which the normals intersect near the center of the region P are more stable against error during grasping because there is (geometrically) more room for error. Since the center of region P is the intersection of the perpendicular bisectors of the edges, the most stable approach to grasping a triangle is to move three fingers to the midpoints of the sides. Although the triangle may move (due to uncertainty in the shape and location of the triangle or location of the fingers), grasping succeeds if errors are sufficiently small. (In actual grasping, the loss of contact of a finger with its side should be easily detectable, and the grasping would be retried.) This procedure does not generally result in a grip which has the fingers at the points where the inscribed circle contacts the edges; however, in the case of extremely obtuse triangles the grip recommended here is more stable against error and in its response to small perturbations than the "circle" grip.

All three fingers could be moved to arrive simultaneously at the midpoints of the sides. If the fingers are able to sense touch they could be stopped at first contact until only one remained moving, and it would make no difference which finger was the moving finger. In any case, as long as the fingers move towards a common point and a different side of the triangle lies between this point and each finger (one finger to a side), a stable grasp is assured. Once the triangle is grasped the positions of the fingers and the ratio of the forces can be used to determine exactly the size and shape of the triangle.

5. Applications to Real Grasping Situations

It is not necessary to use point fingers. If round fingers are used the theory is unchanged if the triangle is increased by moving each side outward by the radius of the finger in contact with it; the fingers are then considered to be points at their centers. However, the original triangle must be used to determine the region P. The fingers need not be entirely round as long as the portion of any finger touching the triangle is round, for then the procedure given above can be used, adjusting the true finger position appropriately from the center of curvature which the theory uses.

The triangle may have mass and friction to the plane with little change to the above theory as long as gravity pulls the triangle perpendicularly against the plane. The main modification is that forces will be felt by the fingers, and if the springs are soft some displacement of the fixed fingers may result. Thus fixed fingers should either be servoed on location during grasping, or be set to be quite stiff if they have programmable compliance. If the moving finger pauses occasionally the time behavior of the forces while the triangle slips should be distinguishable from the constant force which will be observed when an equilibrium grip is achieved. The addition of mass to the triangle means that it is possible for a force sensing finger to observe contact with the triangle.

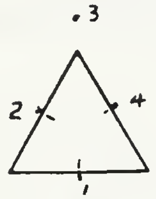
Friction between the triangle and the fingers means that an equilibrium grip can be achieved even when the forces are not perpendicular to the sides. The above theory is not valid under these conditions, but the most stably approached grip is still with the fingers at the midpoints of the sides simply because this allows greater room for positioning error. If a grip is achieved when the forces are not perpendicular to the sides any slipping of the triangle at all three fingers results in a grip closer to the no-friction configuration. Note, however, that with friction it is possible for the triangle to rotate about one of the fixed fingers and be pulled away from contact with the other -- a motion not possible in the no friction case. For example, suppose that in Fig. 2a there is sufficient friction that the triangle does not slip against finger b. The resulting motion would be rotation about finger b, causing the triangle to move away from finger a. If the force applied by each finger is directed between the other two fingers this cannot occur, and any slipping results in a grasp closer to the non-friction grasp.

The theory of grasping a triangle is trivially extended to grasping a convex polygon with three fingers in the frictionless case. Three sides are chosen, and extended to make a triangle. (If two parallel sides are chosen, the procedure is not applicable, and it is well known that three fingers cannot grip a rectangle stably without friction.) The theory is unchanged except that the region P is determined by the actual sides of the polygon, not by their extensions to the triangle. As in the case of the triangle, computation of arcs I and M is usually not needed, since the most stable grasping procedure is to place the fingers on the sides of the polygon in such places as the normals intersect in the center of region P for the polygon sides being used. Should there be a choice as to which sides to use in grasping a polygon, the best choice would appear to be the sides whose region P contains the largest inscribed circle. The same considerations apply to grasping convex portions of more general polygons. However, since inside corners (i.e. where a finger of non-zero radius makes contact with two sides of a polygon) are extremely stable grasping points the theory presented here is probably not optimum for grasping general polygons.

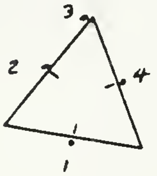
The theory presented here is applicable to most real three or four fingered hands grasping plate-like polygonal objects. The basic ideas require only that two fingers act as very stiff springs and one move in a predetermined direction until grasping forces are established (with suitable adjustment for effects of mass or friction). In hands having programmable compliance in only one direction for each finger the stationary fingers should be made stiff, and the mechanical structure should provide a stiff spring constant in the perpendicular direction. The difficulties suggested in {BFG85] are not likely to arise when small forces are used with stiff springs.

If four fingers are available to grasp a triangle, two fingers on one side of the triangle can be easily counted as a single "virtual" finger in the case of no friction. The force of the virtual finger is the sum of the forces of the paired fingers, and the location is determined by the ratio of the forces (since the moments about the intersection of the normals to the sides of the other two fingers must cancel). By simultaneously adjusting the forces and positions of the paired fingers the physical fingers may move while the virtual finger remains stationary; of course the virtual finger also could move. Fig. 5 illustrates how a triangle can be endlessly rotated by four fingers. The triangle will slip slightly in response to small inaccuracies of the motions, but adjustments will be small and stable. Only one of the fingers need be positioned using a combination of force and position control (counting the two fingers which form the virtual finger as one), and it can be any finger.

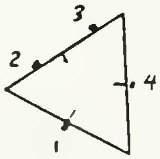
Task: Rotate a triangle in place using three fingers, one of which may be a virtual finger.



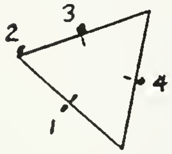
A stable three finger grasp. The numbers show finger location.



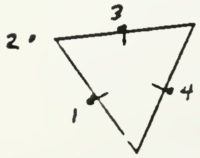
3 just starts to touch, no force applied.



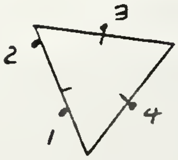
2 and 3 act as a virtual finger, applying together the force which only 2 had been applying, in such a ratio that the virtual finger appears to be at the original point on the triangle. 2 moves away from this point, 3 moves toward it; 1 and 4 continue their simple circular motion.



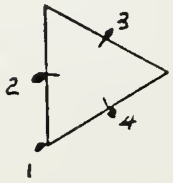
2 is nearly ready to break contact with the edge.



2 prepares to become a virtual finger with 1. 3 and 4 execute simple circular motion.



1 and 2 act as a virtual finger.



Slide has occurred as above, keeping the virtual finger at the same contact point on the triangle. 1 will release and join 4 to become a virtual finger. This is about one quarter of a rotation.

Figure 5. The four finger sliding grasp

6. Appendix: Proofs of theorems

Theorem 1. This follows immediately from the observation that the measure of an angle inscribed in a circle is 1/2 the arc of the circle cut by the angle. The center of the circle of course lies at the intersection of the bisectors of the sides of the angle. The circle is drawn in Fig. 6 for the same configuration of triangle and fingers as in Fig. 4.

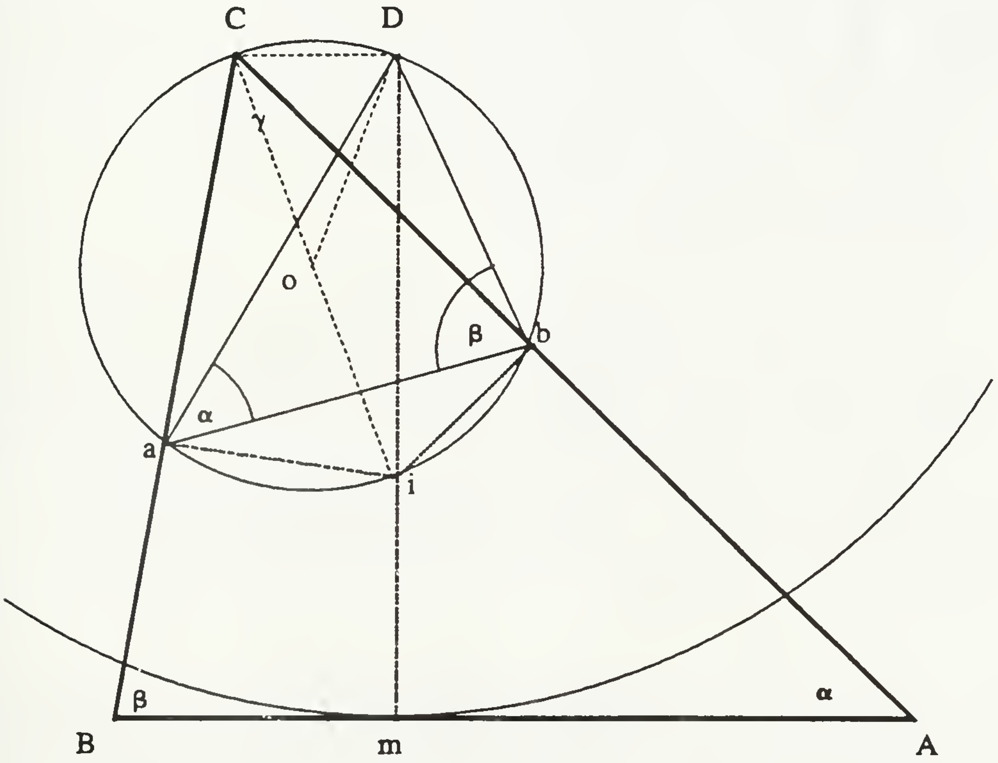


Figure 6. Relations for a triangle held by two fixed fingers a and b

Theorem 2. Draw diameter from C through center o to intersect this circle at i. Observe that angles iaC and ibC are inscribed in semi-circles and thus i is also the intersection of normals to the sides at a and b. From i drop the perpendicular to line AB, defining point m. Extend this line back to where it intersects the circle, and observe that angle iDC is inscribed in a semi-circle, thus CD is parallel to AB. Angle $ACD = \alpha$ (corresponding angles with parallel lines), and cuts the same arc as angle baD. It follows (from arcs or consideration of triangle baD) that the point D is determined by the intersection of the line drawn at a with angle α to line ab and the line drawn at b with angle β to line ab. Thus the location of point D has no connection to where C is; also the distance of m from D is independent of the location of C, being $AC \sin\alpha$, or $BC \sin\beta$. Thus m must lie on a circle of this radius about D.

Theorem 3. Using Fig. 6, let e be the length of side aC, f be the length of side bC, L the distance between the fixed fingers a and b, and r be the distance from C to i. Let δ be angle aCi and ϵ be angle bCi. Then from the law of cosines

$$L^2 = e^2+f^2-2ef\cos\gamma$$

or, rewriting in terms of r

$$L^2 = r^2 (\cos^2\delta + \cos^2\epsilon - 2\cos\delta \cos\epsilon \cos\gamma).$$

Writing γ as the sum of δ and ϵ, this can be written as

$$L^2 = r^2 (\cos\delta \sin\epsilon + \cos\epsilon \sin\delta)^2 = r^2 \sin^2\gamma$$

which means that $r = L/\sin\gamma$. Thus the intersection of the normals to points which are a constant distance apart on two sides of a triangle lies on a circle about the apex of the triangle formed by these two sides. This is circle *I* of Fig. 4; it is not shown in Fig. 6.

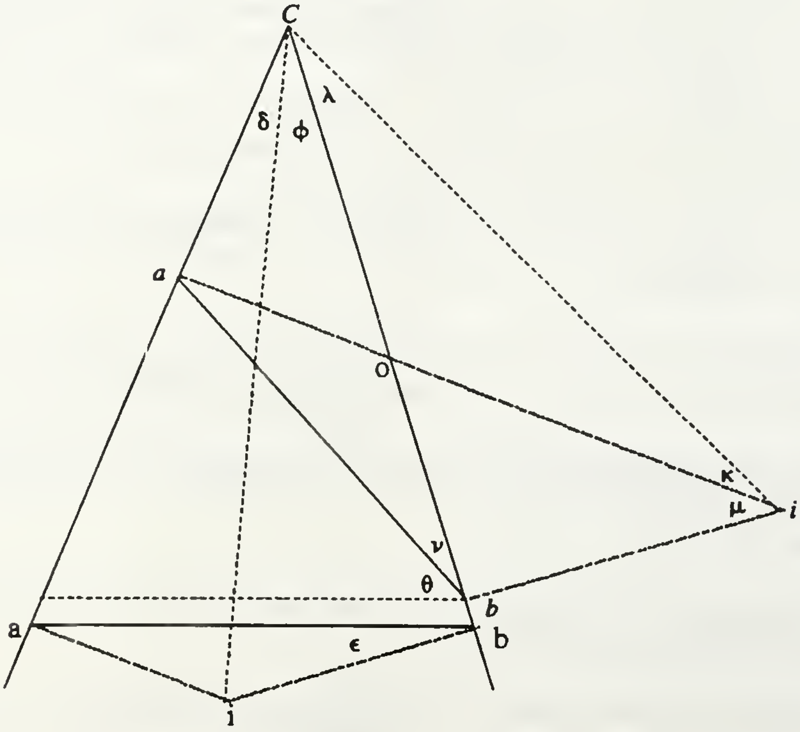


Figure 7. Construction for proof of theorem 4.

Theorem 4. Fig. 7 is drawn in the frame of reference of the triangle to show two different orientations of the triangle in the fixed fingers. The solid lines meeting at C represent the angle of a triangle. The two configurations of fixed fingers are a and b, and *a* and *b*, with normals intersecting at i and *i* respectively. At *b* a line parallel with side ab is drawn, and angles are as indicated. All normals are drawn as dotted, and (from theorem 3) both i and *i* lie on the same circle about C. Angle θ is the rotation of the triangle between the two configurations; it is shown here as the rotation of the fixed fingers in the frame of reference of the triangle. The corresponding motion of i to *i* is the sum $\phi + \lambda$. Observe that

$$\epsilon + \theta + \nu = \kappa + \lambda + \mu$$

since both sides sum to 90 degrees. Further, $\mu = \delta + \phi$ by similar triangles. $\epsilon = \delta$ because both angles cut the same arc of the circle in which they are inscribed (that of theorem 1); similarly $\kappa = \nu$. Making these substitutions results in

$$\theta + \epsilon + \nu = \lambda + \phi + \epsilon + \nu$$

showing that the angle of motion of i to i along the circle about C (i.e. $\lambda + \phi$) is the same as the angle of rotation of the triangle (i.e. θ).

Theorem 5. Observe that the normals to the sides of the triangles must rotate with the triangle; observe that the normal to the third side of the triangle must pass through point D. Therefore, as the triangle rotates by an angle θ the point m will move along the circle M about D by an arc of measure θ.

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